A New Non-Coherent MIMO Scheme: Matrix Coded Modulation "MCM"

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***Abstract*—This paper proposes a new space-time coding scheme for non-coherent MIMO systems. In this scheme, called Matrix Coded Modulation (MCM), a joint channel error-correcting code and space-time code is considered. Coherent systems are those for which Channel State Information (CSI) is available at the transmitters and/or at the receivers, and their performance strongly depend on the channel estimation. Generally, this CSI estimation requires the insertion of pilot-symbols in the transmitted frame which implies a spectral efficiency loss of the global system. The existing non-coherent MIMO systems like Differential Space Time Modulation (DSTM) suffer not only from the degradation of performance compared to coherent systems, but also from many constraints on the channel and the use of memory at reception. In the proposed MCM scheme, decoding can be achieved with or without CSI at the receiving antennas. Moreover, a low-complexity decoding algorithm is described and compared to the existing differential schemes.**

***Keywords:* MIMO systems, coherent, non-coherent, differential coding, error-correcting code.**

I. INTRODUCTION

In the last few years, many techniques of data transmission for wireless Multiple-Input Multiple-Output (MIMO) communication systems have been proposed. Depending on the availability of channel state information at receivers, wireless communication systems can be divided into two categories, coherent and non-coherent. Coherent space-time coding schemes assume a partial or perfect knowledge of CSI at transmitters and/or receivers [7] [10]. This assumption is reasonable when the channel changes slowly compared to the symbol rate, since the transmitter sends training symbols or pilot-frequencies which enable the receiver to estimate the channel accurately. This implies a spectral efficiency loss increasing with the number of antennas. A second disadvantage concerns the performance degradation on fast fading channels due to impaired channel estimation. However, in some situations, we may want to forego channel estimation in order to reduce the cost and complexity of encoding/decoding algorithms at the transmitter/receiver antennas.

When CSI is not available at either the transmitter or at the receiver, several transmission techniques have been proposed. For a Single-Input Single-Output system (SISO), Differential Phase Shift Keying (DPSK) can be applied. To the best of our knowledge, the non-coherent MIMO existing schemes are suitable for Space-Time Block Codes (STBC) [7] [10] or Space-Frequency Block Codes (SFBC) [19]. They can be considered as an extension of the differential techniques used for SISO schemes. Hochwald and Marzetta [14] [11] were the firsts to propose the use of unitary space-time block codes for non-coherent schemes. Hughes in [13] has proposed differential transmit diversity schemes for multiple antenna systems. In some cases, differential STBC techniques, even with an error-correcting code, induce a loss of about $3dB$ compared to the coherent systems [17] [18] [10] with the same error-correcting codes. Most of these proposed Differential STBC are combined with an outer channel error-correcting code in order to improve performance. Decoding can be achieved iteratively on trellis when using a outer convolutional code [16]. On the other hand, the Space-Time Trellis Code (STTC), initially proposed by Tarokh in [8] are considered as an extension of the trellis coded modulation for SISO systems and are dedicated only to coherent detection.

The aim of this paper is to propose a new space-time scheme in which channel error-correcting code and space-time code are combined together and dedicated to non-coherent detection. The remaining parts of this paper are organized as follows: In section II, we remind the Alamouti Differential MIMO scheme. In section III, we introduce the new non-coherent scheme that we call "Matrix Coded Modulation (MCM)" and where space-time code and channel error-correcting code are merged together. In section IV, we present two use cases of MCM. In the first one, we consider the extended Hamming block code $H(n = 8, k = 4, d_{min} = 4)$ as a the channel error-correcting code while in the second one, we consider the Hamming convolutional code obtained by unwrapping the tail-biting trellis of the $H(n = 8, k = 4, d_{min} = 4)$ block code. In section V, we present simulation results, then we compare the MCM scheme and the differential Alamouti 2×2 scheme in terms of performance and complexity. Section VI concludes the paper.

II. DIFFERENTIAL ALAMOUTI SCHEME

Many MIMO Differential schemes exist in the literature and some of them are used, for instance, in the standard IEEE IS-

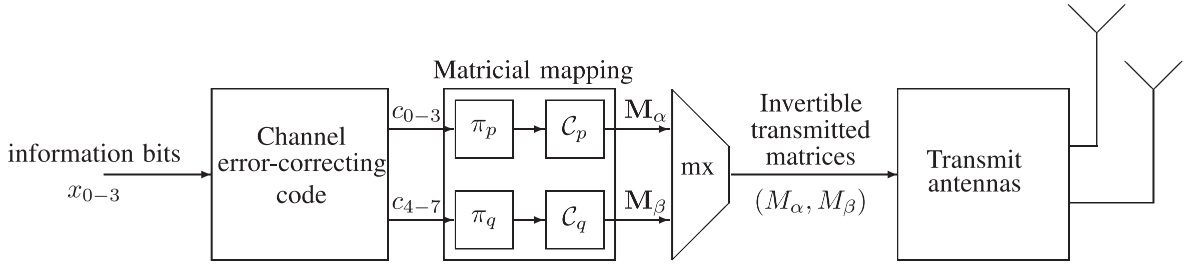

Fig. 1. MIMO-$MCM_{2\times 2}$ system model.

54 [4]. In this section, we remind some necessary preliminaries about the classical differential Alamouti STBC proposed in [12]. We Assume a system consisting of two transmit antennas and one receive antenna. Having the pair of symbols (s_1, s_2) belonging to a $M-PSK$ modulation, the first step consists of computing the differential symbols (A, B) in the orthogonal base $v_1 = (d_1, d_2)$ and $v_2 = (-d_2^*, d_1^*)$ as follows:

$$A = s_1 d_1^* + s_2 d_2^* \quad \text{and} \quad B = -s_1 d_2 + s_2 d_1.$$

The differential space-time encoding can be written as:

$$[s_{2t+1}, s_{2t+2}] = A[s_{2t-1}, s_{2t}] + B[-s_{2t}^*, s_{2t-1}^*]$$

By merging the space-time block encoding and the differential encoding we can write:

$$\begin{aligned} \mathbf{S}_T &= \begin{bmatrix} s_{2t+1} & -s_{2t+2}^* \\ s_{2t+2} & s_{2t+1}^* \end{bmatrix} \\ &= \begin{bmatrix} s_{2t-1} & -s_{2t}^* \\ s_{2t} & s_{2t-1}^* \end{bmatrix} \begin{bmatrix} A & -B^* \\ B & A^* \end{bmatrix} = \mathbf{S}_{T-1}\mathbf{V}_T \end{aligned} \tag{1}$$

where $\mathbf{S}_T$ is the transmitted matrice over the two antennas during 2 symbol-durations T_s, $\mathbf{V}_T$ is unitary matrice verifying $\mathbf{V}\mathbf{V}^H = \mathbf{V}^H\mathbf{V} = \mathbf{I}_2$, H is the hermitian operator " transpose and conjugate " and $\mathbf{I}_2$ the 2×2 identity matrix. At time t, the symbol s_i^t of the matrix $\mathbf{S}_T$ is transmitted over the antenna i. The signal received by antenna j is given by:

$$y_j^t = \sum_{i=1}^{N_t=2} h_{ij}^t s_i^t + n_j^t. \tag{2}$$

where noises n_j^t are modeled as independent samples of a zero-mean complex circularly symmetric Gaussian random variable with variance σ^2, h_{ij}^t is the complex path gain between transmit antenna i and receive antenna j at time t. These coefficients are modeled as independent samples of a complex circularly symmetric Gaussian random variable with zero mean and variance of 0.5 per dimension. The fading is assumed to be constant over a frame of length L message bits and varies from one frame to another. Information matrices $\mathbf{S}_T$ are transmitted over a wireless communication channel and the received matrix $\mathbf{Y}_T$ is given by:

$$\mathbf{Y}_T = \mathbf{H}_T\mathbf{S}_T + \mathbf{N}_T \tag{3}$$

where $\mathbf{H}_T$ is the $N_t \times N_r$ channel matrix and $\mathbf{N}_T$ is $N_r \times 1$ the noise matrix.
From Eq.(1), and for $\mathbf{H}_T = \mathbf{H}_{T-1}$, we express the received matrix $\mathbf{Y}_T$ at time T in function of the previous received matrix $\mathbf{Y}_{T-1}$:

$$\mathbf{Y}_T = \mathbf{Y}_{T-1}\mathbf{V}_T + (\mathbf{N}_T - \mathbf{N}_{T-1}\mathbf{V}_T) \tag{4}$$

A Conventional Detector (CD) can be used to estimate the useful transmitted information matrix $\hat{\mathbf{V}}_T$.

$$\hat{\mathbf{V}}_T = Arg \max_{(\hat{\mathbf{V}}_T)} \mathcal{R}]\{Tr[\tilde{\mathbf{V}}_T^{\mathbf{H}}(\mathbf{Y}_{T-1})^H\mathbf{Y}_T]\} \tag{5}$$

Many other differential detectors can be used like the Decision Feedback Differential Detection (DFDD) [18] but in all these differential schemes the main constraint is to consider a quasi-static channel over a frame of L transmitted matrices.

III. MCM SYSTEM MODEL

In this section, we introduce the general principle of the MCM technique dedicated to non-coherent systems. Information bits are encoded with a channel error-correcting code and then divided into streams to be mapped directly into matrices of complex symbols without the use of classical mapping from binary into m-ary (m being the number of bits per symbol). In Fig.1, we give an example of an MCM scheme dedicated to a 2×2 non-coherent MIMO system. Information bits, x_{0-3}, two streams of coded bits c_{0-3} and c_{4-7}. These two streams are interleaved with (π_p, π_q) and mapped directly into a pair of invertible matrices $(\mathbf{M}_\alpha, \mathbf{M}_\beta)$ of size $N_t \times T$ each to be consecutively transmitted over the N_t antennas. These invertible matrices should be chosen from a multiplicative group $\mathcal{G}$ such that:
$(\mathbf{M}_\alpha, \mathbf{M}_\beta) \in (\mathcal{C}_p, \mathcal{C}_q)$ where $(\mathcal{C}_p, \mathcal{C}_q)$ are two different cosets of $\mathcal{G}$.
The choice of (π_p, π_q) and $(\mathcal{C}_p, \mathcal{C}_q)$ is not arbitrary. In fact, the considered choice should introduce a relation between the consecutively transmitted matrices $\mathbf{M}_\alpha$ and $\mathbf{M}_\beta$ that depends on the employed group $\mathcal{G}$ and the channel-error correcting code. This new approach based on a joint space-time coding and channel coding will be detailed through examples in section IV. The simple detection algorithm is based on the relationship between the pair of matrices $(\mathbf{M}_\alpha, \mathbf{M}_\beta)$ and can be performed without any channel information. The fading is

assumed to be constant over a frame of length L information bits. At the receiver, detection is achieved without any channel estimates and is based on the relation between the consecutively transmitted matrices.

IV. MCM WITH BLOCK AND CONVOLUTIONAL CHANNEL ERROR-CORRECTING CODE

An advantage of the MCM scheme is its ability to be adapted for both block or convolutional codes. In this section, we illustrate the MCM scheme by presenting it in 2 models with the Hamming block and convolutional code. Although this MCM scheme can be generalized for any systems of $N_t \times N_r$ antennas, we present these 2 models of the MCM scheme for 2×2 systems.

A. MCM with a Hamming block code

This simplified MCM model consists of $N_t = 2$ transmit antennas and $N_r = 2$ receive antennas, with a small error-correcting block code: the systematic Hamming H$(8, 4, 4)$ of rate $r = k/n = 1/2$ where $n = 8$ is the codeword length and $d_{min} = 4$ its minimum Hamming distance. A block of $k = 4$ bits $x = (x_0, x_1, x_2, x_3)$ are encoded with a H$(8, 4, 4)$. Each codeword $c = (c0, c1, ...c_7)$ is generated as $c = x \cdot \mathbf{G}$ where $\mathbf{G}$ is equal to:

$$\mathbf{G} = \begin{bmatrix} 1 & 0 & 0 & 0 & 1 & 1 & 1 & 0 \\ 0 & 1 & 0 & 0 & 0 & 1 & 1 & 1 \\ 0 & 0 & 1 & 0 & 1 & 0 & 1 & 1 \\ 0 & 0 & 0 & 1 & 1 & 1 & 0 & 1 \end{bmatrix}$$

The encoded bits $(c0, c1, ...c_7)$ are mapped directly into a pair of $N_t \times T$ matrices $(\mathbf{M}_\alpha, \mathbf{M}_\beta)$. These matrices are chosen from the multiplicative group $\mathcal{G}_w$ of Weyl [3] which is very simply generated as the set of 12 cosets $(\mathcal{C}_0, \mathcal{C}_1, \ldots, \mathcal{C}_{11})$ each containing 16 invertible matrices. The first coset $\mathcal{C}_0$ is defined as:

$$\mathcal{C}_0 = \left\{ \alpha \begin{bmatrix} 1 & 0 \\ 0 & \pm 1 \end{bmatrix} \quad , \quad \alpha \begin{bmatrix} 0 & 1 \\ \pm 1 & 0 \end{bmatrix} \right\}$$

with $\alpha \in \{+1, -1, +i, -i\}$. The 12 cosets of $\mathcal{G}_w$ are derived from $\mathcal{C}_0$ as follows:

$$\mathcal{C}_k \ = \ \mathbf{a}_k \cdot \mathcal{C}_0 \quad \forall\ k = 0, 1, \ldots, 11$$

where the matrices $\mathbf{a}_0, \mathbf{a}_1, \ldots, \mathbf{a}_5$ are respectively:

$$\mathbf{a}_0 = \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix}, \mathbf{a}_1 = \begin{bmatrix} 1 & 0 \\ 0 & i \end{bmatrix}, \qquad \mathbf{a}_2 = \tfrac{1}{\sqrt{2}} \begin{bmatrix} 1 & 1 \\ 1 & -1 \end{bmatrix},$$
$$\mathbf{a}_3 = \tfrac{1}{\sqrt{2}} \begin{bmatrix} 1 & 1 \\ i & -i \end{bmatrix}, \mathbf{a}_4 = \tfrac{1}{\sqrt{2}} \begin{bmatrix} 1 & i \\ 1 & -i \end{bmatrix}, \mathbf{a}_5 = \tfrac{1}{\sqrt{2}} \begin{bmatrix} 1 & i \\ i & i \end{bmatrix}.$$

and the matrices $\mathbf{a}_6, \mathbf{a}_7, \ldots, \mathbf{a}_{11}$ are given by: $\mathbf{a}_{k+6} = \eta \mathbf{a}_k$, with $\eta = (1+i)/\sqrt{2}$ $\forall k = 0, 1, \ldots, 5$.
At time t the symbol s_i^t of the matrix $\mathbf{M}_a$ is transmitted over the antenna i. In our example of the MCM system, as the dimensions of the matrices are $N_t \times T = 2 \times 2$, matrix $\mathbf{M}_a$ is transmitted in 2 symbol-durations T_s. Writing in matrix form, we obtain:

$$\mathbf{Y}_T = \mathbf{H}_T \mathbf{M}_\alpha + \mathbf{N}_T \tag{6}$$

$\mathbf{Y}_T$ is the received matrix during 2 symbol-durations T_s on the 2 antennas between instants T and $(T + 2T_s)$. We assume a uniform power allocation at the transmission in order to maintain a constant radiated power on the average of a space-time codeword duration. The extended Hamming $H(8, 4, 4)$ block code is systematic. The 4 useful information bits are permuted with π_0 and then mapped into the coset $\mathcal{C}_p$. Similarly the 4 redundant bits are permuted with π_2 and then mapped into the coset $\mathcal{C}_q$. Having 16 possible codewords and 16 matrices in each coset, then for any codeword $(c_0, c_1, ..., c_7)$ generated by the H$(8, 4, 4)$, there is a unique couple of matrices $(\mathbf{M}_a, \mathbf{M}_b) \in \ \mathcal{C}_p \times \mathcal{C}_q$ which verifies the equation below :

$$\mathbf{M}_\alpha \cdot \mathbf{M}_a^{-1} - \mathbf{M}_\beta \cdot \mathbf{M}_b^{-1} = 0 \tag{7}$$

where $(\mathbf{M}_\alpha, \mathbf{M}_\beta) \in \ \mathcal{C}_p \times \mathcal{C}_q$ are the transmitted matrices. The Eq.7 has a unique solution:

$$(\mathbf{M}_a, \mathbf{M}_b) = (\mathbf{M}_\alpha, \mathbf{M}_\beta) \quad \in \quad \mathcal{C}_p \times \mathcal{C}_q \tag{8}$$

The choice of the 2 interleavers (π_0, π_2) and the 2 cosets $(\mathcal{C}_p, \mathcal{C}_q)$ is not arbitrary. One of the possible solutions verifying Eq.(7) is the two cosets $(\mathcal{C}_0, \mathcal{C}_2)$ and the two permutations $\pi_0 \ : \ (0, 1, 2, 3) \ \rightarrow \ (0, 1, 2, 3)$ and $\pi_2 \ : \ (0, 1, 2, 3) \ \rightarrow \ (0, 3, 2, 1)$. It was obtained by an exhaustive computing search. In this case, the constellation of the modulation (i.e. the possible complex values of the coefficients' matrices) is $\{\pm 1, \pm i, (\pm 1 \pm i)/\sqrt{2} \quad \cup \quad 0\}$ which is noted 4-QAM $\cup$ 0. Taking the eight coded bits $c \ = \ (c_0, c_1, \cdots, c_7)$ and given the two cosets $(\mathcal{C}_0, \mathcal{C}_2)$ and the pair of interleavers (π_0, π_2), the MCM encoder selects the pair of matrices $(\mathbf{M}_\alpha, \mathbf{M}_\beta)$ among the pair of cosets$(\mathcal{C}_0, \mathcal{C}_2)$ according to a specific mapping rule. For the codeword 00011101 we compute $(i_1 = 2^0 \times\ 0\ +\ 2^1\ \times\ 0\ \times\ 2^2\ \times\ 0\ +\ 2^3\ \times\ 1\ =\ 8\ )$ and $(i_2\ =\ 2^0\ \times\ 1\ +\ 2^3\ \times\ 1\ +\ 2^2\ \times\ 0\ +\ 2^1\ \times\ 1\ =\ 11\ )$ then the pair of matrices assigned to this codeword will be: $M_{i_1} = M_\alpha = \begin{bmatrix} 0 & 1 \\ 1 & 0 \end{bmatrix}$ and $M_{i_2} = M_\beta = \frac{1}{\sqrt{2}} \begin{bmatrix} 1 & -1 \\ -1 & -1 \end{bmatrix}$

The 2×2 matrices $\mathbf{M}_\alpha$ and $\mathbf{M}_\beta$ are transmitted consecutively on the 2 antennas during $4T_s$. Signals arriving at the 2 receive antennas undergo independent fading and can be expressed as follows:

$$\mathbf{Y}_T \quad = \quad \mathbf{H}_T \mathbf{M}_\alpha + \mathbf{N}_T \tag{9}$$
$$\mathbf{Y}_{T+1} \quad = \quad \mathbf{H}_{T+1} \mathbf{M}_\beta + \mathbf{N}_{T+1} \tag{10}$$

Assuming a constant block fading channel during $4T_s$ ($\mathbf{H}_T \ = \ \mathbf{H}_{T+1}$), and with the unicity of solution in Eq.(7) the implementation of the decoding algorithm consists of finding the couple $(\hat{M}_a, \hat{M}_b)$ solution of the following minimization:

$$(\hat{\mathbf{M}}_a, \hat{\mathbf{M}}_b) = Arg \min_{(\mathbf{M}_a, \mathbf{M}_b)} ||\mathbf{Y}_T \mathbf{M}_a^{-1} - \mathbf{Y}_{T+1} \mathbf{M}_b^{-1}|| \tag{11}$$

where $||\mathbf{X}||$, the Hilbert norm, is equal to $Trace(\mathbf{X}\mathbf{X}^H)$. With the bijective relation between a codeword c and a couple $(\mathbf{M}_\alpha, \mathbf{M}_\beta)$ we can then provide the 8 "best" coded bits of c and then the "best" 4 information bits. The $H(8, 4, 4)$

has a weak error-correction capability. We note that our new construction with the matrices of the Weyl group is not similar with to a space-time code with linear dispersion code (LD) [15]. The LD codes are based on the optimisation of matrices in order to maximise the capacity and diversity gains whereas our new construction is based on a novel detection criteria based on invertibility of the employed matrices.

B. MCM with a Hamming convolutional code

a-Emission

We introduce below an example of a2×2 -MCM with a very small convolutional error-correcting code built by unwrapping the 4-states "tail-biting" or circular trellis of the Hamming code $(n = 8, k = 4, d_{min} = 4)$. This small convolutional code is shown in Fig. 2.

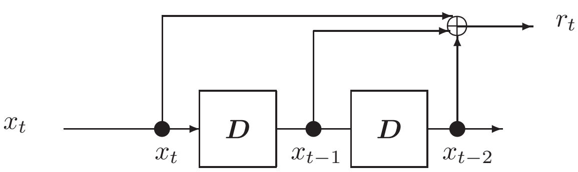

Fig. 2. Hamming convolutional 2-bits state encoder

The future goal of our research is to extend the MCM scheme to be adapted with Turbo codes [5] which have two simple component trellis. The convolutional MCM scheme uses the same structure as our MCM block scheme with the same group of matrices $\mathcal{G}_w$ used in Example 1. The main difference being that the useful information bits are now presented in sequence $(\ldots, x_{t-1}, x_t, x_{t+1}, \ldots)$ and are encoded to produce a sequence of redundant bits $(\ldots, r_{t-1}, r_t, r_{t+1}, \ldots)$. In order to do the matricial mapping, information and redundant bits are grouped by paquets of 4 bits such as$(x_t, r_t, x_{t+1}, r_{t+1})$. Encoding and decoding algorithms are done on a 4-state trellis with branches labeled by 4 bits as shown in Fig. 3. Similarly, we choose the pair of cosets $(\mathcal{C}_0, \mathcal{C}_2)$ and the permutations $\pi_0 : \ (0,1,2,3) \rightarrow (0,1,2,3)$ and $\pi_2 : \ (0,1,2,3) \rightarrow (0,3,2,1)$. Having 16 matrices in each coset and 16 possible combinations of 4 bits, each trellis section is a complete bipartite graph. Each group of 4 bits on a branch of the trellis has its proper corresponding matrix in the appropriate coset. Matrices are selected alternatively in cosets $\mathcal{C}_0$ and $\mathcal{C}_2$ and then they are transmitted serially on the 2 antennas $(\ldots, \mathbf{M}_{T-1}, \mathbf{M}_T, \mathbf{M}_{T+1}, \ldots) \in \ldots \times \mathcal{C}_0 \times \mathcal{C}_2 \times \mathcal{C}_0 \times \ldots$.

b-Reception

Fig. 4 explains the decoding algorithm of the MCM convolutional scheme. We use a variant of the Viterbi algorithm [1] by modifying the metric computation on each branch of the treillis such as:

$$\gamma_T(\mathbf{M}_b) = \min_{(\mathbf{M}_a, \mathbf{M}_c)} \{ (\lambda \| \mathbf{Y}_{T-1}\mathbf{M}_a{}^{-1} - 2\mathbf{Y}_T\mathbf{M}_b{}^{-1} + \mathbf{Y}_{T+1}\mathbf{M}_c{}^{-1} \| + \mu \| \mathbf{Y}_T - \hat{\mathbf{H}}'_T \mathbf{M}_b \|) \} \quad (12)$$

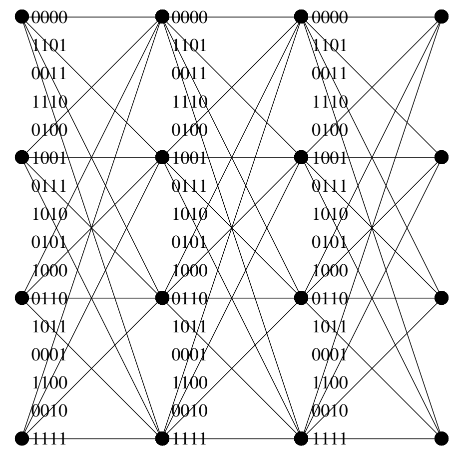

Fig. 3. 4-state trellis of the convolutional Hamming code.

λ and μ are adaptive weights over iterations to merge the minimization of the channel variations and the minimization of the euclidean distance between received and transmitted signals. When no CSI is available at the receivers, $(\lambda, \mu) = (1, 0)$. Iterative decoding with an appropriate channel estimation corresponds to $(\lambda, \mu) = (p, 1-p)$ with $0 \leq p \leq 1$. The estimated values $\hat{H}'_t$ are the estimations of the channel matrix associated with each branch of the trellis and they are given by:

$$\hat{\mathbf{H}}'_T(\hat{\mathbf{M}}_b) = (\mathbf{Y}_{T-1}\hat{\mathbf{M}}_a{}^{-1} + 2\mathbf{Y}_T\hat{\mathbf{M}}_b{}^{-1} + \mathbf{Y}_{T+1}\hat{\mathbf{M}}_c{}^{-1})/5 \quad (13)$$

After evaluating the metrics of the branches $\gamma_T(\mathbf{M}_b)$, we compute the metric states classically as in the Viterbi algorithm:

$$\Gamma(s_T) = \min_{\mathbf{M}_b} (\Gamma(s_{T-1}) + \gamma_T(\mathbf{M}_b)) \quad (14)$$

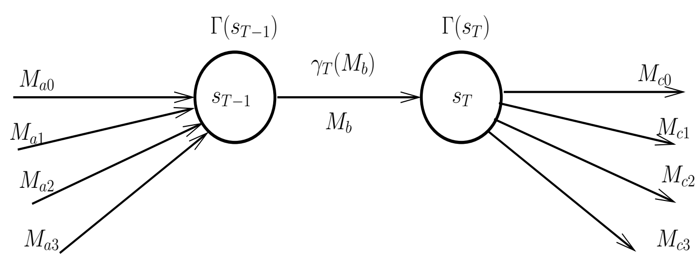

Fig. 4. Schematic of computing label paths of the MCM convolutional decoding algorithm

V. Simulations and results

In this section, we present some simulation results in terms of Bit Error Rate (BER) versus the Energy-per-Bit to Noise ratio $(Eb/N0)$ for different 2×2-MIMO schemes. We assume a block fading channel quasi-static on a frame of L message bits and varying independently from one frame to another. Fig.5 represents the performance results for the differential Alamouti scheme concatenated with the Hamming block code $H(8,4,4)$ compared to our MCM scheme with the same error-correcting code as proposed in section IV-A. As a reference, we compare the non-coherent scheme with the coherent Alamouti 2×2 scheme with the same Hamming block code. The differential Alamouti scheme induce a loss about 4 dB at $BER = 10^{-3}$ compared to the coherent Alamouti scheme with the same error-correcting code. We remind that our goal is to find new non-coherent scheme where the loss and the detection complexity between coherent and non-coherent scheme can be reduced.

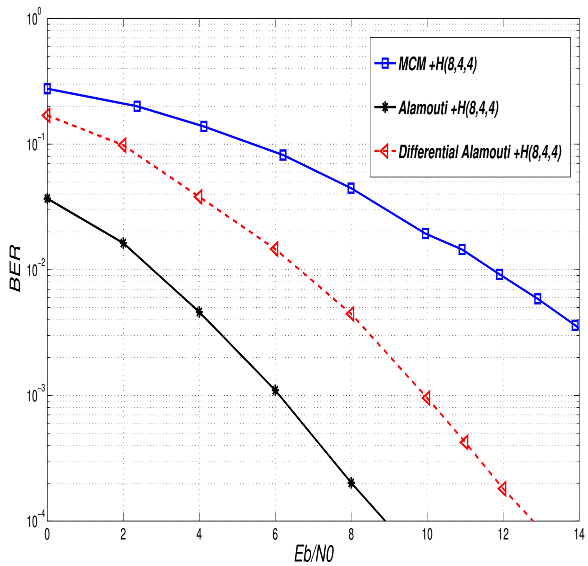

Fig. 5. MCM with Hamming $(8,4,4)$ error-correcting code, L=128

At first glance, we can see that the performance of the MCM scheme is far from that of the differential Alamouti scheme. Nevertheless, computing the number of operations (additions and multiplications) respectively needed to decode the MCM schemes and the differential Alamouti schemes gives the advantage for the first one (see Table I) in terms of complexity. To simplify our study we use the 2×2 scheme with the Hamming $(8,4,4)$ code but we can also generalize it for other convolutional codes. Table I shows that the MCM scheme is 8 times less complex than the differential scheme. The loss of diversity gain between the Alamouti scheme and the MCM scheme is due to the construction of the invertible matrices of the Weyl group.

TABLE I
COMPLEXITY TABLES

	nb of additions	nb of multiplications
Differential Alamouti	1536	1536
MCM block scheme	128	192

The polynomial distribution of the Euclidean distances based on the detection criteria of Eq.11 when using the matrices of the Weyl group and the permutations given in example 1 of section IV is given by:

$$D_{Hblock}(x) = 1 + 14x^4 + x^8 \qquad (15)$$

This polynomial distribution of the Euclidean distances also represents that of the H(8, 4, 4) based on the Hamming distance which is an important result.

Fig.6 shows the improvement of performance results for the differential Alamouti scheme concatenated with a convolutional code compared to our MCM scheme described in section IV-B always for $L = 128$. An advantage of the MCM scheme is its ability to combine the space-time encoder with a convolutional channel encoder and so to use an iterative receiver.
The theoretical calculation of the polynomial distribution of the Euclidean distances referring to the metric in Eq.14 gives the follows :

$$D_{Hconv}(x) = 1 + 2x^{12} + x^{20} \qquad (16)$$

The minimal Hamming distance was 4 in Eq.15 for the MCM block scheme while it is 12 in Eq.16 which is also a promising result especially if we use another convolutional channel error-correcting code suitable for the MCM scheme.
The MCM convolutional scheme with the Hamming convolutional code does not perform well on the first iteration $(\lambda,\mu) = (1,0)$. But the second iteration with $(\lambda,\mu) = (0.5,0.5)$ improves the performance and an important gain of about $6dB$ at $BER = 10^{-3}$ appears. The most important gain compared to the differential scheme is given by the third iteration with $(\lambda,\mu) = (0,1)$. We can see that the MCM convolutional scheme is better than the differential Alamouti scheme with a gain of $0.7dB$ at $BER = 10^{-3}$. Also, the non-coherent MCM scheme tends to its lower bound with perfect knowledge of CSI with a loss of about $2dB$ at $BER = 10^{-3}$ only. However, the differential Alamouti scheme introduces a loss of about $4dB$ at $BER = 10^{-3}$ compared to its lower bound which is the coherent Alamouti scheme. We also notice that the MCM scheme in the coherent case induces a loss of about $0.5dB$ at $BER = 10^{-3}$ compared to the Alamouti coherent scheme. These results show the forcefulness of the MCM iterative scheme in both coherent and non-coherent context. Currently, we are working on generalizing the MCM scheme with the Golay convolutional code with its unwrapped minimal tail-biting trellis. Its construction is being under study. Using an appropriate sets of cosets, permutations and another group of invertible matrices will clearly improve the performance.

VI. Conclusions and Perspectives

In this paper, we have described a new MIMO coding scheme called Matrix Coded Modulation "MCM", for transmitting data over wireless communication channels with very low decoding complexity for non-coherent systems. This new

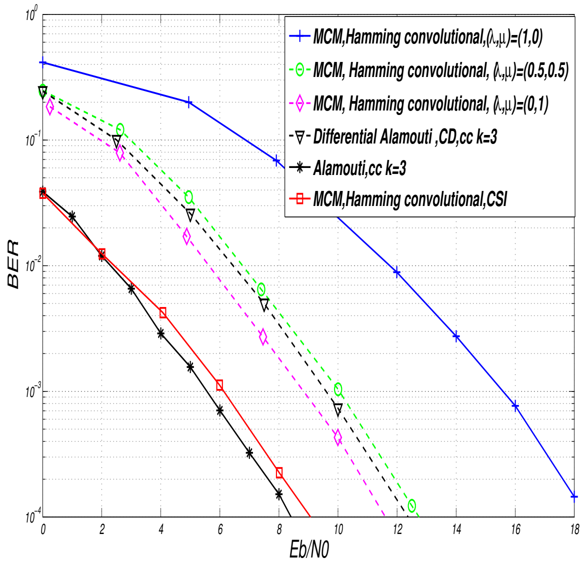

Fig. 6. MCM with Hamming convolutional error-correcting code, L=128

scheme presents a novel concept based on a joint channel-coding, modulation and space-time coding which is different from that of the classical schemes where the channel error-correcting code and the STBC are serially concatenated. The MCM scheme seems to be an attractive competitor of the Alamouti scheme especially when it will be used with a good error-correcting code and appropriate cosets and permutations. In non-coherent context, the performance of this scheme is not far from the existing ones such as the differential Alamouti scheme, but the complexity of the proposed scheme is largely reduced. Also a theoretical study of the MCM schemes based on the calculation of the polynomial distribution of the detection criteria was achieved. This calcultaion give promising results allowing . The application of this MCM scheme would be very interesting especially when used with an efficient channel error-correcting code with a higher minimum Hamming distance d_{min} like the Golay convolutional code [9] or a Turbo code [5]. This new scheme can be generalized for any convolutional code and any higher order modulations and it seems that it may outperform the existing differential schemes. The main goal of this research is to gain partially and asymptotically the performance degradation of non-coherent schemes compared to coherent ones but without any CSI at the receivers and assuming a slow varying wireless channel.

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